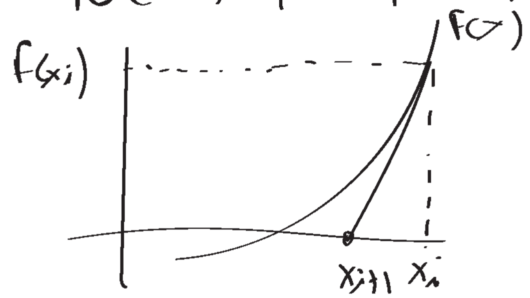
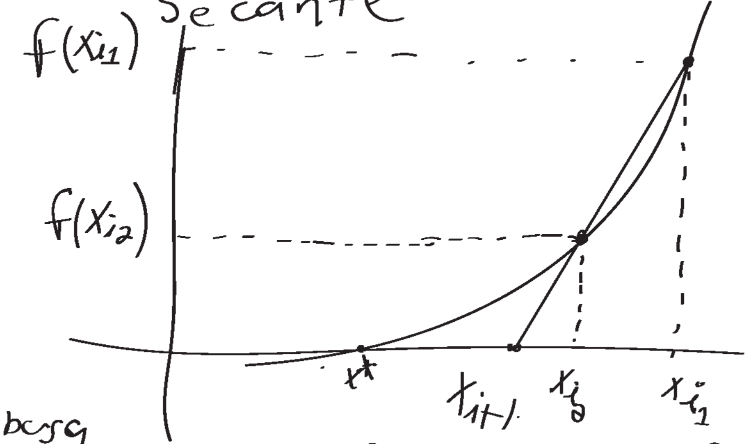
En el método de la secante se necesitan más puntos que en el método Newton-Raphson

Newton Raphson



Secante

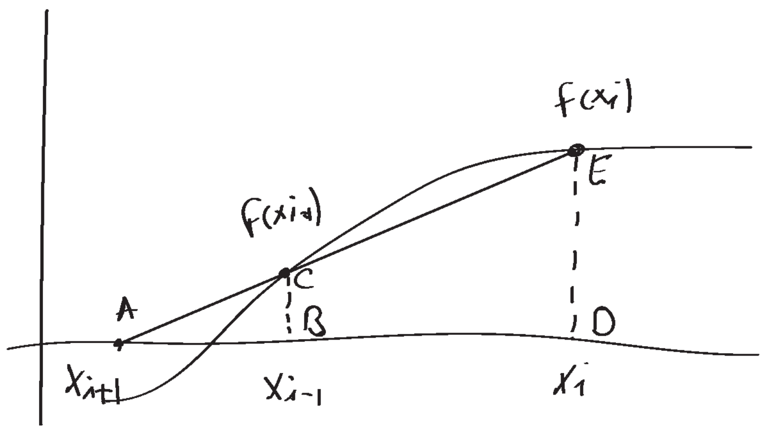


El primer enfoque se basa en sustituir una aproximación de Newton-Raphson

$$X_{k+1} = X_k - \frac{f(x)}{f'(x)} \qquad f'(x) \approx \frac{f(X_i) - f(X_{i-1})}{X_i - X_{i-1}}$$

$$x_{k+1} = X_k - \frac{f(x)(X_k - X_{k-1})}{f(X_k) - f(X_{k-1})}$$

El otro punto para deducir es directamente derivando de la gráfica



El triangulo $\triangle ABC$ y $\triangle ADE$ son similares

$$\frac{ED}{CB} = \frac{AD}{AB}$$

relacionando

$ED = f(x_i)$ $AD = x_i - x_{i+1}$

$CB = f(x_{i-1})$ $AB = x_{i-1} - x_{i+1}$

$$\frac{f(x_i)}{f(x_{i-1})} = \frac{x_i - x_{i+1}}{x_{i-1} - x_{i+1}}$$

$$\frac{f(x_i)}{f(x_{i-1})} = \frac{x_i - x_{i+1}}{x_{i-1} - x_{i+1}}$$

$$f(x)(x_{i-1} - x_{i+1}) = f(x_{i-1})(x_i - x_{i+1})$$

$$(x_{i-1})f(x) - (x_{i+1})f(x) = x_i f(x_{i-1}) - (x_{i+1})f(x_{i-1})$$

sustituyendo

$$(x_{i+1})f(x_{i-1}) - (x_{i+1})f(x) = x_i f(x_{i-1}) - (x_{i-1})f(x)$$

$$x_{i+1}\left(f(x_{i-1}) - f(x_i)\right) = x_i f(x_{i-1}) - (x_{i-1})f(x_i)$$

$$\boxed{x_{i+1} = \frac{x_i f(x_{i-1}) - (x_{i-1})f(x_i)}{f(x_{i-1}) - f(x)}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})} \qquad x_i - x_{i+1} = \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

$$f(x_i)(x_i - x_{i-1}) = (x_i - x_{i+1})\left(f(x_i) - f(x_{i-1})\right)$$

$$f(x_i)(x_i - x_{i-1}) = (x_i - x_{i+1})f(x_i) - (x_i - x_{i+1})f(x_{i-1})$$

$$f(x_i)(x_i - x_{i-1}) - f(x_i)(x_i - x_{i+1}) = -(x_i - x_{i+1})f(x_{i-1})$$

$$f(x_i)(x_i - x_{i-1} - x_i + x_{i+1}) = -(x_i - x_{i+1})f(x_{i-1})$$

$$\frac{f(x_i)}{f(x_{i-1})} = \frac{-(x_i - x_{i+1})}{(x_{i+1} - x_{i-1})} = \frac{x_i - x_{i+1}}{x_{i-1} - x_{i+1}}$$