Ejercicio. Encuentre el gradiente y la matriz Hessiana de la función

$$f(x) = x_1^2 + 2x_1x_2 + 3x_2^2 + 4x_3^2 - 5x_2x_3$$

También encuentre la derivada direccional de la función en el punto (1,1,1) en la dirección

$$d = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_3} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

$$\frac{\partial f(x)}{\partial x_1} = 2x_1 + 2x_2$$

$$\frac{\partial f(x)}{\partial x_2} = 2x_1 + 6x_2 - 5x_3$$

$$\frac{\partial f(x)}{\partial x_3} = 8x_3 - 5x_2$$

$$\nabla f = \begin{bmatrix} 2x_1 + 2x_2 \\ 2x_1 + 6x_2 - 5x_3 \\ 8x_3 - 5x_2 \end{bmatrix}$$

$$\frac{\partial^2 f(x)}{\partial x_1 x_1} = \frac{\partial}{\partial x_1}\left(\frac{\partial f(x)}{\partial x_1}\right) =$$

$$\nabla f(x)^T \mathbf{u}$$

$$[\mathbf{H}] = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} & \frac{\partial^2 f}{\partial x_1 x_3} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 x_3} \\ \frac{\partial^2 f}{\partial x_3 x_1} & \frac{\partial^2 f}{\partial x_3 x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} 2x_1 + 2x_2 \\ 2x_1 + 6x_2 - 5x_3 \\ 8x_3 - 5x_2 \end{bmatrix}$$

$$[\mathbf{H}] = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} & \frac{\partial^2 f}{\partial x_1 x_3} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 x_3} \\ \frac{\partial^2 f}{\partial x_3 x_1} & \frac{\partial^2 f}{\partial x_3 x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$$\frac{\partial^2 f(x)}{\partial x_1 x_1} = \frac{\partial}{\partial x_1}\left(\frac{\partial f(x)}{\partial x_1}\right) = \frac{\partial}{\partial x_1}(2x_1 + 2x_2) = 2$$

$$\frac{\partial^2 f(x)}{\partial x_2^2} = \frac{\partial}{\partial x_2}\left(\frac{\partial f}{\partial x_2}\right) = \frac{\partial}{\partial x_2}(\cancel{2x_1} + 6x_2 - \cancel{5x_3}) = 6$$

$$\frac{\partial^2 f(x)}{\partial x_3^2} = \frac{\partial}{\partial x_3}\left(\frac{\partial f}{\partial x_3}\right) = \frac{\partial}{\partial x_3}(8x_3 - \cancel{5x_2}) = 8$$

$$\frac{\partial^2 f(x)}{\partial x_1 x_2} = \frac{\partial}{\partial x_1}\left(\frac{\partial f}{\partial x_2}\right) = \frac{\partial}{\partial x_1}(2x_1 + \cancel{6x_2} - \cancel{5x_3}) = 2$$

$$\frac{\partial^2 f(x)}{\partial x_1 x_2} = \frac{\partial^2 f(x)}{\partial x_2 x_1} = \frac{\partial}{\partial x_2}\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial}{\partial x_2}(\cancel{2x_1} + 2x_2) = 2$$

$$\frac{\partial^2 f(x)}{\partial x_3 x_1} = \frac{\partial^2 f(x)}{\partial x_1 \partial x_3} = \frac{\partial}{\partial x_3}(2x_1 + 2x_2) = 0$$

$$\frac{\partial^2 f(x)}{\partial x_2 x_3} = \frac{\partial^2 f(x)}{\partial x_2 x_3} = \frac{\partial}{x_2}(\cancel{8x_3} - 5x_2) = -5$$

$$H = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 6 & -5 \\ 0 & -5 & 8 \end{bmatrix}$$

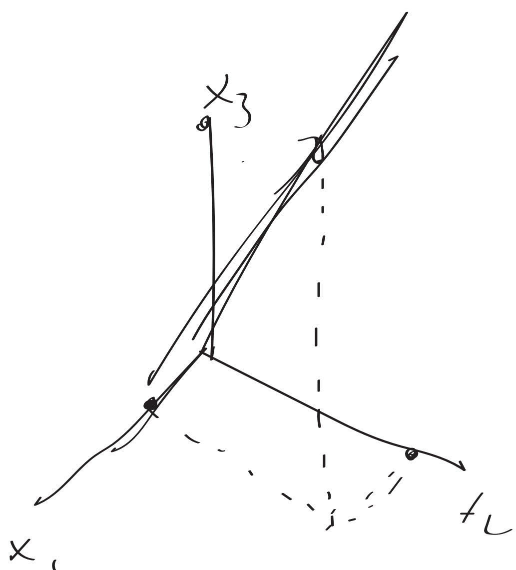
$$\nabla f = \begin{bmatrix} 2x_1 + 2x_2 \\ 2x_1 + 6x_2 - 5x_3 \\ 8x_3 - 5x_2 \end{bmatrix}$$

$$d = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\nabla f(1,1,1) = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

$P(1, 1, 1)$

$$\begin{array}{cccc} d\,(& 1, & 2, & 3\,) \\ & x_1 & x_2 & x_3 \end{array}$$



$$D_a = \nabla f(y)^T u$$

$$u = \frac{d}{\|d\|} = \frac{1}{\sqrt{1^2+2^2+3^2}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$$

$1 \times 3 \qquad 3 \times 1$

$$D_a = [4, 3, 3] \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix} = \frac{4+6+9}{\sqrt{14}} = \frac{19}{\sqrt{14}}$$